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THE
ELEMENTS
OF
DESCRIPTIVE GEOMETRY

BY

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PREFACE

DESCRIPTIVE GEOMETRY contains the rudiments of Mechanical Drawing, Shades and Shadows, Perspective, and Advanced Technical Drawing. Drawing is a language by means of which we can inform one another of the shape and size of different objects. In teaching a child to read, we do not, at first, give him a book of Shakespeare, but start him with the alphabet and words of one syllable. Consequently, the author has endeavored to build up the subject step by step, inserting the necessary definitions only as needed, rather than by starting the student with innumerable definitions, which, at the time, mean little or nothing to him. In the drawings, too, the author has employed the synthetical method by first presenting a drawing of the given lines, then a drawing illustrating the next step in combination with the preceding one, and so on by a series of drawings, completing the example with the finished drawing.

Intersections, developments, etc., are usually included in the regular course in Mechanical Drawing. It seems needless, therefore, to include them in this elementary text. If it is desirable to include more advanced work in the course of Descriptive Geometry, this book will form such a foundation that the student may readily interpret any higher treatise.

H. C. B.

CHESTER, PA., August, 1914.

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It will be noticed that the plates are assembled at the back of the book in the order above given, and that each plate when unfolded will extend beyond the text, so that the drawings of one plate, as well as any particular page of the text, may be visible at the same time.

The drawings are numbered according to the article to which they refer, and are lettered in accordance with the sequence in which they occur in each article.

THE ELEMENTS OF DESCRIPTIVE GEOMETRY

1. Definition of Descriptive Geometry.

Descriptive Geometry is the language of drawing. It treats of the methods of representing objects and of solving problems relating thereto.

2. Projection of a Point.

If, from a point in space, a perpendicular (⊥) is let fall to some plane in space, the point where this ⊥ intersects the plane is the *projection* of the point on that plane.

2a. This ⊥ is called the *projecting line*, and the plane is called the *plane of projection.*

3. Orthographic Projection.

In this system of projection, the position of the point in space is determined by its projections on Horizontal and Vertical Co-ordinate Planes of Projection. Since the planes are either horizontal or vertical, all projecting lines are vertical or horizontal respectively. (Cf. Art. 2.)

3a. The projection of a point on a Horizontal Plane (H) is called its H projection; for instance,

a^H or a,* equals the H projection of point A. Likewise, its projection on V is called its V projection; for instance, a^V or a',* equals V projection of point A.

4. Ground Line.

The intersection of H and V is a line and is called the Ground Line (GL).

5. The Four Quadrants.

H and V are considered to be indefinite in extent and will intersect in GL to form four quadrants, called arbitrarily, first, second, third, and fourth quadrant. These are shown in Fig. 5a. (Plate I.)

The first quadrant is *above* H and *before* V.

The second quadrant is *above* H and *behind* V.

The third quadrant is *below* H and *behind* V.

The fourth quadrant is *below* H and *before* V.

5a. If we look at Fig. 5a, from the right end, we can see the quadrants a little more clearly. See also Fig. 5b.

5b. In order to represent a point on a piece of paper (which is a plane), we must imagine *H to be revolved* as shown by the arrows in Figs. 5a and 5b.

5c. Fig. 5c (first quadrant) is part of Fig. 5a and shows the projections of a point, A, upon H and V.

5d. Rotating H of Fig. 5c, as shown, we have Fig. 5d.

---

* Both forms are in use. Throughout this text, we shall use a^H and a^V.

Note that a^H (or a)* and a^V (or a′)* are in a line $\perp$ to GL.

5e. Likewise Figs. 5e, 5g, 5i represent the second quadrant, third quadrant, and fourth quadrant, respectively, each being part of Fig. 5a, and rotating H in each, as shown, we have Figs. 5f, 5h, and 5j.

5f. In the actual drawing, we might combine Figs. 5d, 5f, 5h, and 5j to form Fig. 5k. (Plate II.)

5g. Special Note. Rule for Plotting.†

ABOVE *use* ^{V}
BEHIND *use* ^{H}
G————L
BELOW *use* ^{V}
BEFORE *use* ^{H}

Explanation. If a point is *above,* it is shown above the GL and lettered with a "V." If a point is *before,* it is shown below the GL and lettered with an "H."

6. Profile Plane.

Let us cut the co-ordinate planes (Art. 3) by a plane $\perp$ to H and V, as shown in Fig. 6a. (Plate II.) Such a plane is called a *profile plane* (*P*).

6a. Revolving the H Plane, as shown, we have Fig. 6b.

7. Notation and Plotting.

Any point in space is either to the left of P or to the right of P. For, as shown in Fig. 6a (Plate II), E is to the left of P, and, therefore, when revolved as in Fig. 6b, it is shown to the left of P. Likewise,

---

* See note, page 10.

† This rule probably originated with Tracy and North.

F is to the right of P and, therefore, is shown to the right of P in Fig. 6b. The distance, x (Fig. 6b), will be the ⊥ distance from E to the plane P and equal to Ee^P in Fig. 6a. Similarly, y (Fig. 6b) equals Ff^P. (Fig. 6a.)

7a. Notation with Respect to H and V.

1. If a point A is *above** H, it will be written A = 6a,† which means, A is 6″ above H. (See Fig. 5c [Plate I], $Aa^H = 6''$; or Fig. 5d, $oa^V = 6''$.)

2. If a point C is 5″ *below** H, it will be written C = 5l.† (See Fig. 5g, $Cc^H = 5''$; or Fig. 5h, $oc^V = 5''$.)

3. If a point A is 4″ *before** V, it will be written A = 4f.† (See Fig. 5c, $Aa^V = 4''$; or Fig. 5d, $oa^H = 4''$.)

4. If a point C is 6″ *behind** V, it will be written C = 6h.† (See Fig. 5g, $Cc^V = 6''$; or Fig. 5h, $oc^H = 6''$.)

7b. Notation with Respect to P.

1. If a point E is $3\frac{1}{2}''$ to the *left* of P, it will be written E = $3\frac{1}{2}$L. (See Fig. 6a [Plate II], $Ee^P = 3\frac{1}{2}$; or Fig. 6b, $x = 3\frac{1}{2}$.)

2. If a point F is $2\frac{1}{4}''$ to the *right* of P, it will be written F = $2\frac{1}{4}$R. (See Fig. 6a, $Ff^P = 2\frac{1}{4}$; or Fig. 6b, $y = 2\frac{1}{4}$.)

---

* Some books use + for above and − for below, + for before and − for behind, it being understood that above or below precedes before or behind, or the opposite.

† **Hint.** Associate a with *a*bove, l with *l*ow (below), f with *f*ore (before), h with *h*ind (behind). This system of notation will be used throughout this text.

7c. Complete Notation.

In Fig. 7a (Plate II), A = 5L, 2a,† 3f, or 5L, 2 above, 3 before.

B = 2L, 4a,† $1\frac{1}{2}$h, or 2L, 4 above, $1\frac{1}{2}$ behind.

C = $1\frac{1}{2}$R, 2l,† 2h, or $1\frac{1}{2}$R, 2 below, 2 behind.

D = 7R, 3l,† 2f, or 7R, 3 below, 2 before.

8. Profile Projection.

For the present, P will be taken to the right of all points in space.

Similar to Art. 2, a line through a point $\perp$ to P is a *projecting line*, and its intersection with P is the P projection of that point.

Similar to Art. 3, the projection of a point on P is called the P projection and written a^P, b^P, c^P, etc.

Figs. 8a and 8b (Plate II) represent the P projection of a point A in the first quadrant, 8″ above H and 4″ before V.

9. Combination of H, V, and P Projections. Fig. 9a (Plate III).

In Art. 8, we learned to plot the P projection of a point, the distances from H and V being given. In this article, we shall be able to plot the P projection directly from the H and V projections of a point, and vice versa.

1. Given a point A = 30″L, 8″a, 4″f. (Fig. 9a, Plate III.)

---

† See note, page 12.

Plot H and V projections as usual. (Arts. 5 and 7.)

Required: P projection of A.

Draw XY at 45°.

Draw horizontal line through a^H to 45° line XY, then project up or down to line drawn horizontally through a^V.

a^P is the required P projection.

2. Given same point A. (Fig. 9a.)

Plot P projection as usual. (See Art. 8.)

Required: H and V projections.

Draw horizontal line through a^P.

Draw vertical line through a^P to XY, then draw horizontal line.

Stop these horizontal lines at required distance from P (not a^P), as in this case, 30″ to the left of P. This gives required projections, the first horizontal line giving a^V, and the second, a^H.

Note. GL and HH are usually made a continuous line.

9a. Other Methods of Finding P Projection.

1. Through H projection (b^H, Fig. 9b), draw line (b^Hx) horizontally to P, then draw (xy) at an angle of 45° to H; then draw (yb^P) up or down to horizontal line (b^Vb^P) through V projection. This (b^P) is the P projection. But note, if H projection is *behind*, draw 45° line in direction indicated by arrow xy; and if H projection is *before*, draw in direction indicated by arrow zw.

2. Through H projection (b^H, Fig. 9c), draw line (b^Hx) horizontally to P, then draw arc (x–x'–x''–y) in *clockwise* direction with O as center to H, and then draw (yb^P) up or down to horizontal line (b^Vb^P) through V projection. This (b^P) is the required P projection.

10. Projections of Straight (Right) Lines.

A line is determined by two points — say, A and B. (Fig. 10a, Plate III.) Consequently, the H projection of a line AB is determined by the H projection of A and B, *i. e.*, join a^H and b^H (Figs. 10a, 10b), and the V projection of a line is determined by the V projection of A and of B; *i. e.*, join a^V and b^V (Figs. 10a, 10b), and the P projection of a line is determined by the P projections of A and B; *i. e.*, join a^P and b^P (Fig. 10c).

10a. Special Notes about Lines.

1. Lines are assumed to be indefinite in extent.

2. Lines are inclined to, parallel to, or perpendicular to the co-ordinate planes H, V, or P.

3. If two lines are parallel in space, their projections upon each of the co-ordinate planes will be parallel, for their *projecting planes* (cf. Art. 11) are parallel.

4. If two lines are $\perp$ in space, their projections will not generally be $\perp$, unless one line is parallel to a co-ordinate plane. (Cf. Art. 57.)

5. Lines parallel to H (or V) will have their

V (or H) projection parallel to GL and will appear as their actual or " true length " in their H (or V) projection. Fig. 10e represents a line inclined to H and parallel to V, the H projection (a^Hb^H) being parallel to GL, and the V projection (a^Vb^V) being equal to the true length.

6. Lines perpendicular to V (or H) will appear as a point on V (or H), while the other projection will be perpendicular to the GL and will also show the true length of the line. Fig. 10f represents a line ⊥ to H, the H projection will appear as a point (a^Hb^H), and the V projection (a^Vb^V) will be ⊥ to GL and will be equal to the true length of the line.

11. Projecting Plane. Fig. 10a (Plate III).

The plane passing through a line and ⊥ to H, V, or P is called the H, V, or P *projecting plane* of that line. In Fig. 10a, ABb^Ha^H is the H projecting plane of AB, and ABb^Va^V is the V projecting plane of AB.

12. To Assume a Line. Fig. 12a (Plate III).

Draw the projections at random, *i. e.*, draw a^Hb^H and a^Vb^V in any direction. Of course, a^H and a^V, likewise b^H and b^V, must be in the same line ⊥ to GL. (Cf. Art. 5d.)

12a. To Assume a Point on a Line. Fig. 12b (Plate III).

Take any point, c^V, on the proper projection,

a^Vb^V, and project it vertically until it intersects the other projection, a^Hb^H. This gives the other projection, c^H.

13. Two Intersecting Lines.

If two lines intersect in space, the point where their V projections intersect must be directly above or below (in same line $\perp$ to GL) the point where their H projections intersect.

This is true, because two lines intersect in a point. Now, a point has its two projections in a line $\perp$ to GL. (Art. 5d.) The V projection of this point must lie in the V projections of both lines and is, therefore, at the intersection of V projections of both lines. Likewise, the H projection of this point is at the intersection of H projections of both lines. Fig. 13a (Plate III) shows two intersecting lines, AB and CD.

Note that e^V, where a^Vb^V intersects c^Vd^V, and e^H, where a^Hb^H intersects c^Hd^H, are in same line $\perp$ to GL.

13a. If the point where the V projections of the two lines intersect is in the same horizontal line in which their P projections intersect, the two lines intersect in space. Fig. 13b (Plate IV) shows this clearly.

14. To Assume Two Intersecting Lines. Figs. 13a or 13b (Plates III and IV).

Assume any line, AB, by drawing $a^H b^H$ and $a^V b^V$ at random. (Art. 12.) Of course, a^V and a^H as well as b^V and b^H must be in line ⊥ to GL.

Draw one projection, $c^V d^V$, of CD intersecting corresponding projection, $a^V b^V$, of AB in some point as e^V.

Find e^H. Drop ⊥ from e^V to $a^H b^H$.

Through e^H draw any line, $c^H d^H$, intersecting $a^H b^H$. Of course c^H and c^V as well as d^H and d^V must be in line ⊥ to GL.

These two lines, AB and CD, intersect in space.

15. Planes.

Since the projection of a plane is a plane and hence could not be projected upon a piece of paper without covering the entire drawing, it is customary, in drawing planes, to represent them by the lines in which they intersect or pierce the co-ordinate planes. These lines are called *traces*.

The line in which a plane intersects H is called the H Trace (HT).

The line in which a plane intersects V is called the V Trace (VT).

The line in which a plane intersects P is called the P Trace (PT).

Planes are assumed to be indefinite in extent, hence their traces may be continued as desired.

Fig. 15a (Plate IV) represents a pictorial drawing of a plane S in the first quadrant. When H is revolved (Fig. 15b) we have the usual way of representing a plane.

15a. Planes are designated in various ways. A capital letter will be placed at the intersection of the two traces, and, when this is not feasible, *i. e.*, when the traces intersect off the paper, the capital letter will be enclosed in parentheses on each trace. Figs. 20a, 20b, 20c, 20d, 20e, and 20f indicate planes A, B, C, D, E, and F, respectively.

15b. In order to further describe a plane, some method of locating the traces must be used. The intersection of the traces will be indicated by its distance (to the right or left) from P. (Cf. Art. 7b.) The angles which the traces make with the GL will be measured according to the principles of trigonometry, using the GL as the initial line and measuring all angles counter-clockwise, *i. e.*, opposite to the direction in which the hands of a clock revolve. Fig. 15b shows the plane S = 2L, V120°, H225°, which means that the vertex, S, is 2″ to the left of P; that the angle x, between the GL and VT, equals 120°; and that the angle y between GL and HT equals 225°.

When the traces are parallel to the GL, or if one trace is missing, a somewhat different method of locating the plane will be necessary. Fig. 20d shows plane D where the VT is 2½″ above the GL and the HT is 2″ before the GL. This will be indicated by D = M, 2½a, 2f, the M meaning that the regular value is missing. Fig. 20e shows plane E with the VT 1½" above the GL. This will be indicated by E=M, 1½a, M. Fig. 20f will be indicated by F=M, M, 2h.

16. Traces of a Plane Meet in GL.

If either the HT or the VT cuts the GL, the other trace must cut the GL in the same point, otherwise we would have a straight line (GL) intersecting a plane in two points.

17. Corollary to Art. 16.

If either trace is parallel to the GL, the other trace must be parallel to GL or be missing.

18. Determination of Planes.

A plane is determined by (1) three points not in a straight line; (2) two lines either intersecting or parallel; (3) a line and a point (not in the line).

19.* 1. Determination of a Plane by Three Points.

Fig. 19a (Plate IV) shows the common method of assuming a plane by three points. While the points might have been assumed in other positions, in general one point, A, is assumed in GL; one point, B, in H; and one point, C, in V.

2a. Determination of a Plane by Two Intersecting Lines.

Fig. 19a also shows the common method of assuming a plane by two intersecting lines. In general, the lines are assumed to intersect in the GL. (See Art. 14.) One line, AB, is assumed in H; and one line, AC, in V.

---

* May be omitted at first.

2b. Determination of a Plane by Two Parallel Lines.

Fig. 19b shows the common method of assuming a plane by two parallel lines. In general, one line, AB, is assumed in V parallel to the GL; and the other, CD, in H, parallel to GL.

3. Determination of a Plane by a Line and Point.

Fig. 19c shows some of the various ways of solving this problem. For instance, we may assume a line, AC, in V, and a point, B, in H; or assume a line, AB, in H, and a point, C, in V; or any line, BC, oblique to H and V and a point, A, in GL.

20. The Seven Ways of Representing a Plane.

The seven ways of representing a plane are shown in Figs. 20a, b, c, d, e, f, g (Plate IV).

21. To Assume a Plane.

Fig. 21a (Plate IV). (May be omitted if Art. 19 is taken.)

Draw two lines, VT and HT, intersecting in GL. Call one VT. Call other HT. Plane A is then assumed.

22. To Find PT of a Plane.

Given VT and HT of plane A. Fig. 22a1 (Plate IV).

To find the PT.

Continue HT to P. This gives x^H. x^V will be in the GL. (Fig. 22a2.)

Find P projection of this point, *i. e.*, find x^P. (Fig. 22a2.)

Continue VT to P. This gives y^V. y^H will be in the GL. (Fig. 22a3.)

Find P projection of this point, *i. e.*, find y^P. (Fig. 22a3.)

Join these two P projections, x^P and y^P. (Fig. 22a.)

$x^P y^P$ is required PT.

23. Traces of Lines and Planes.

The place where a line, or plane, pierces or intersects a plane is called the *trace* of a line or a plane.

The trace of a line is a point, because the intersection of a line and a plane is a point.

The trace of a plane is a line, because the intersection of two planes is a line.

The H Trace (HT) is where the H plane is pierced.

The V Trace (VT) is where the V plane is pierced.

The P Trace (TP) is where the P plane is pierced.

23a. Trace of a Line.

Figs. 23a, 23b, and 23b1 (Plate V) show that the line AB pierces H at $y^H = HT$.

Figs. 23a, 23b, and 23b2 show that the line AB pierces V at $x^V = VT$.

Figs. 23b and 23b3 show that the line AB pierces P at U $(u^V, u^H) = PT$.

THEORY

H Trace. V projection of HT is in GL, also in V projection of given line, and, therefore, at their intersection.

H projection of HT is in ⊥ through V projection and also in H projection of given line, and, therefore, at their intersection.

V Trace. H projection of VT is in GL, also in H projection of given line, and, therefore, at their intersection.

V projection of VT is in ⊥ through H projection and also in V projection of given line, and, therefore, at their intersection.

P Trace. V projection of PT is in P and also in V projection of given line, and, therefore, at their intersection.

H projection of PT is in P and also in H projection of given line, and, therefore, at their intersection.

RULE

HT. Continue V projection to GL, then up or down to H projection (continued if necessary).

VT. Continue H projection to GL, then up or down to V projection (continued if necessary).

PT. Continue H and V projections to Profile plane.

EXPLANATION OF RULE

Given the line AB. To find its traces.

HT. In Fig. 23b1, continue the V projection (a^Vb^V) of the line AB to the GL, giving y^V, then draw y^Vy^H vertically until it intersects the H projection (a^Hb^H) continued, at y^H. $y^H = HT$.

VT. In Fig. 23b2, continue the H projection (a^Hb^H) of the line AB to the GL, giving x^H, then draw x^Hx^V vertically until it intersects the V projection (a^Vb^V) continued, at x^V. $x^V = VT$.

PT. In Fig. 23b3, continue the H projection (a^Hb^H) of the line AB to the P plane, giving u^H, also continue the V projection (a^Vb^V) to the P plane, giving u^V. The point U, shown by the two projections u^H and u^V, is the PT.

Figs. 23b1, 23b2, and 23b3 are combined to form Fig. 23b.

Fig. 23a is a pictorial drawing of the HT and VT of AB.

23b. P Projection of Traces of a Line.

Fig. 23c (Plate V) shows the three projections of the line CD. Notice that the P projection shows clearly the HT at W, the VT at Z, and that the PT is shown clearly by the H and V projections.

24. To Assume a Line in a Plane.

THEORY

If a line lies in a plane, the HT of the line must be in the HT of the plane, and the VT of the line must be in the VT of the plane, and the PT of the line must be in the PT of the plane.

CONSTRUCTION

Given plane A. Fig. 24a (Plate V). To assume a line in plane A.

Assume any point, A, shown by $a^V a^H$, in VT.

Assume any point, B, shown by $b^V b^H$, in HT.

Join A and B, *i. e.*, join a^V and b^V; a^H and b^H.

AB is the required line.

Note. While the above is the most common method, a point might have been assumed in PT, and the other point in either VT or HT.

24a. Another Method for Art. 24.

THEORY

If a line is assumed parallel to the HT (or VT), its V (or H) projection will appear parallel to the GL, while its H (or V) projection will appear parallel to the H (or V) Trace. See Art. 10a5.

CONSTRUCTION

In Fig. 24b, given the plane B. Assume any line, CD, parallel to the HT, *i. e.*, draw $c^H d^H$ parallel to HT, continue $c^H d^H$ to GL, then draw $c^H c^V$ up or down to the VT and draw $c^V d^V$ parallel to the GL.

Note that the VT of the line is in the VT of the plane. See Art. 24.

25. To Assume a Point on a Plane.

Assume a line on the plane. (Art. 24.)

Assume a point on the line. (Art. 12a.)

26. To Revolve a Point in Space about an Axis ⊥ to One Co-ordinate Plane.

THEORY

As the axis is ⊥ to H, V, or P, on one plane, we shall see a circle as the path of the point, while the other projection of the path of the point will be in a line ⊥ to that projection of the axis which is shown in its true length.

CONSTRUCTION

In Fig. 26a (Plate V), Pictorial Representation, and Fig. 26b1, Orthographic Projection, we are to revolve the point A about an axis CD, CD being ⊥ to V.

With c^Vd^V, Figs. 26a and 26b, as a center (always take that projection of the axis as the center where the projection of the axis appears as a point), and with a radius c^Vd^V to a^V (if the center is a V [or H] projection, use the V [or H] projection of A), describe an arc, as shown, in which a^V will move.

At the same time, a^H will move ⊥ to c^Hd^H; in this case, parallel to the GL.

At any particular time, a^V and a^H must be in a straight line ⊥ to GL.

Similarly, it may be worked out for an axis ⊥ to H or P.

Of course, the point may be in any quadrant, while the line may be in the same or a different quadrant.

27. To Revolve a Point into One of the Coordinate Planes (about a line in that plane of projection).

In Figs. 27a (Plate V), Pictorial Drawing, and 27b1, given the line AB, lying in H, and a point C, in space.

To revolve C about AB into H.

THEORY

Fig. 27a. Drop a ⊥ CO from C to AB.

With O as the center, and a radius of CO, revolve C into H.

The new position of C is C^{RH}.

Note that $oc^{RH} = OC = \sqrt{oc^{H^2} + Cc^{H^2}}$ (in a right triangle),

$$= \sqrt{oc^{H^2} + xc^{V^2}}.$$

CONSTRUCTION

Through c^H (if the line is in H [or V], use the H [or V] projection of the point) draw c^Ho ⊥ to the corresponding projection of the line, *i. e.*, a^Hb^H. (Fig. 27b2.)

Then off on the side of the paper (Fig. 27c), construct a right triangle in order to find the radius mentioned under **theory**, making $rs = xc^V$, $st = oc^H$, then $rt = oc^{RH}$.

Lay off $oc^{RH} = rt$ (measuring from the line and on the ⊥). (Fig. 27b2.)

Then $c^{RH} = C$ revolved into H.

The completed drawing is shown in Fig. 27b.

28. To Find the True Length of a Line, or the distance between two points (applying Art. 27).

Figs. 28a, 28b, 28c (Plate VI).

Analysis 1. Given the line AB. (Fig. 28a1.) To find true length of AB.

Revolve A about a^Hb^H as an axis into H. (Fig. 28a2.) In this case, $st = 0$, $rs = xa^V$, and, therefore, $rt = rs = a^Ha^{RH}$, as in Fig. 27c.

Likewise, revolve B about a^Hb^H as an axis into H. (Fig. 28a, $b^Hb^{RH} = yb^V$.)

Then $a^{RH}b^{RH}$ = the true length of the line AB.

Note that the problem might have been done by revolving the points about a^Vb^V as an axis into V.

Analysis 2. Revolve the H (or V) projecting plane (Art. 11) of the line about its H (or V) T, *i. e.*, about a^Hb^H (or a^Vb^V), as an axis into H (or V).

This is represented in Fig. 28b (Pictorial Drawing), and Fig. 28c (Orthographic Projection).

28a. To Find Angle between a Line and the Co-ordinate Planes.

Find the angle between AB and the co-ordinate planes. (Fig. 28a.)

Continue a^Hb^H and $a^{RH}b^{RH}$ until they intersect. This forms the angle between AB and the H plane.

Likewise, continue a^Vb^V and $a^{RV}b^{RV}$ (see Note, Art. 28, Analysis 1) until they intersect.

This forms the angle between AB and V.

28b. Special Cases of Art. 28.

The student can easily see the application of Art. 28 to the problems in Figs. 28d and 28e.

29. To Find the True Length of a Line (applying Art. 26).

ANALYSIS

If we revolve one end of the line about an axis ⊥ to H (or V) through the other end of the line until the line is parallel to V (or H), the line will appear in its true length. (Art. 10a5.)

CONSTRUCTION

Given AB. Fig. 29a1 (Plate VI).

To find its true length.

Take one end, B, of the line AB and revolve it about an axis, ⊥ to H, through A, until the line is parallel to V. (The H projection of B must move in a circular arc with a^H as a center and a^Hb^H as a radius until a^Hb^H is parallel to the GL [Fig. 29a2]; the V projection of B will move in a straight line parallel to the GL, and its position at any time will be found by dropping a perpendicular from b^{HR} [Fig. 29a]. See Art. 26.)

The V projection of AB, thus revolved (a^Vb^{VR}), will be its true length.

The angle between AB and H will be the angle $ma^V b^{VR}$, shown on Fig. 29a.

29a. Special Cases of Art. 29. (Fig. 29b.)

See note under Art. 28a.

30. Corollary to Art. 29. May also apply to Art. 28.

Given a line AB . . . .″ long, making . . . .° with H (or V). To find its projections, knowing the angle its H (or V) projection makes with GL.

In this case, AB = 4″ and makes an angle of 45° with H, while its H projection makes an angle of 30° with GL.

In Fig. 30a (Plate VI), take $a^H x^H$ (any length) making the required angle (30°)* with the GL. Revolve $a^H x^H$ to the position $a^H x^{HR}$ and drop a perpendicular, $x^{HR} x^{VR}$, to a (45°)* line through a^V. $a^V x^{VR}$ is the true length of a line meeting all conditions except that of length. On $a^V x^{VR}$, measure the required length, (4″)* = $a^V b^{VR}$. Then revolve this point, b^{VR}, back, giving b^V and b^H. Then AB is the line required.

31. To Pass a Plane through a Line.

ANALYSIS AND CONSTRUCTION

Each trace of the plane must pass through the corresponding trace of the line.

Given the line AB. Fig. 31a1 (Plate VI).

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* Use values given in the problem.

To pass a plane through AB.

Find the HT, D, of the line. Find the VT, C, of the line. (Art. 23.)

The HT of the plane must pass through D, and the VT of the plane must pass through C. As it takes more than two points to determine a plane (Art. 18), a number of solutions is possible.

Draw any two lines through C and D, intersecting the GL in any point as A. (Fig. 31a.)

Then $Ad^H = HT$ and $Ac^V = VT$ of the required plane.

32. To Pass a Plane through Two Lines.

32a. Case 1. When the lines are intersecting.

Find the HT of one line. (Art. 23a.)

Find the HT of the other line.

The line joining the two points, *i. e.*, the HTs just found, will be the HT of the required plane. (Cf. Art. 31.)

In a similar manner, find the VT of each line. The line joining them will be the required VT of the plane.

Note that the HT and the VT meet in the GL. (Art. 16.)

32b. Case 2. When the lines are parallel.

The principle is the same as in Art. 32a.

In Fig. 32a (Plate VI), AB and CD are the parallel lines and T is the resulting plane.

This problem is especially useful in Perspective, where the perspective of a line is found by means of the intersection of the Picture Plane and a Visual Plane through the line.

32c. Special Case. When the traces of the lines are off the paper.

If the given lines do not cut the GL within the limits of the paper, new lines intersecting the given lines must be assumed and Art. 32a applied to them.

In Fig. 32b1, the given lines are AB and CD, while the new lines are FK, determined by the point F on CD and the point G on AB, and IK, determined by the point I on AB and the point J on CD. (Figs. 32b2, 32b3, 32b.)

33. To Pass a Plane through Three Points.

Join the points so as to form two intersecting lines (Art. 14) and apply Art. 32a.

34. To Pass a Plane through a Line and a Point.

From the point, draw a line intersecting the given line (Art. 14) and apply Art. 32a.

35. Miscellaneous Principles of Planes and Lines.

a. If two planes are parallel, their traces on the same plane will be parallel. (Intersection of one plane with two parallel planes.)

b. If two planes are perpendicular, their traces on the same plane will *not* generally be perpendicular, except when one plane is perpendicular to a co-ordinate plane.

c. If a line is parallel to a plane, its projections need *not* necessarily be parallel to the corresponding traces of the plane.

d. If a line is perpendicular to a plane, its projections will be perpendicular to the corresponding traces of the plane, for —

1. The line is perpendicular to the plane.

2. The H projecting plane is perpendicular to the plane. (Two planes are perpendicular, if one plane contains a line perpendicular to the second plane.)

3. But the H projecting plane is perpendicular to H.

4. Therefore, the H projecting plane is perpendicular to the intersection of the given plane and H, *i. e.*, to the HT of the plane. (A plane perpendicular to two planes will be perpendicular to the intersection of the two planes.)

5. Hence the H projection of the line is perpendicular to the HT. (Any line of a plane cutting the line of intersection of two planes will be perpendicular to the line of intersection.)

e. To draw a line through a point perpendicular to a plane.

Given point A and the plane A in Fig. 35a (Plate VI).

Through A, draw the projections of the required line perpendicular to the corresponding traces of the plane; *i. e.*, draw $a^V b^V \perp$ to VT and $a^H b^H \perp$ to HT.

Note that the point B is not necessarily the point where the line pierces the plane. (Cf. Art. 51.)

36. To Pass a Plane through a Point Parallel to a Given Plane.

In Fig. 36a (Plate VII), Plane A and the point A are given, while B = the required plane.

ANALYSIS AND CONSTRUCTION

1. Through the point A pass a line AY parallel to a line (the HT or the VT being preferable) in the plane. That is, through a^V draw $a^V y^V$ parallel to VT of A, and $a^H y^H$ parallel to GL. (Art. 10a3.)

2. Through this line, AY, pass the plane B parallel to the given plane A. This is done by

a. Finding one trace, say HT at y^H, of the line AY. (Art. 23a.)

b. Passing one trace, HT, of the plane B through this point, y^H.

c. Drawing traces of the required plane parallel to the given plane. (Art. 35a.)

d. Noting that the VT and HT of B intersect in the GL. (Art. 16.)

37. To Pass a Plane through a Point Perpendicular to a Given Plane.

In Fig. 37a (Plate VII), the plane A and the point A are given, while B is the required plane.

ANALYSIS AND CONSTRUCTION

1. Through the point A pass a line AX perpendicular to the given plane A; *i. e.*, through a^V draw a^Vx^V ⊥ to VT of A, and through a^H draw a^Hx^H ⊥ to HT of A.

2. This line, AX, lies in the required plane, B.

3. The HT and the VT of this line, AX, will be in the HT and VT of the required plane B; *i. e.*, HT of B must pass through x^H, and VT of B must pass through y^V.

4. Now, since an indefinite number of planes may be passed through a point perpendicular to a plane (notice in 1 and 2 that we have only one line, AX, which by Art. 18 is not sufficient to determine a plane), we may draw HT and VT of B in any direction, so long as the traces meet in the GL (Art. 16); *i. e.*, draw HT of B through x^H to z (any point in GL); then draw VT of B through z and y^V.

38. To Pass a Plane through a Line Parallel to a Given Plane.

This is generally impossible, except as shown in Art. 36, where the given line is parallel to either trace of the given plane.

39. To Pass a Plane through a Line Perpendicular to a Given Plane.

In Fig. 39a1 (Plate VII), given the line AB and the plane A.

1. Through any point, C, of the line AB, draw a line, CD, perpendicular to the given plane A. (Fig. 39a.) (Cf. Art. 35e.)

2. A plane, B, passing through these two lines, AB and CD, will meet the requirements, for —

a. Two intersecting lines determine a plane. (Art. 18.)

b. The plane B contains the given line AB.

c. The plane B will be perpendicular to the given plane, A, as it contains a line, CD, perpendicular to the given plane.

3. Complete by Art. 32 (pass a plane through two intersecting lines).

Note that the VT of A is not necessarily perpendicular to the VT of B, and HT of A is not necessarily perpendicular to HT of B. (Art. 35b.)

40. To Pass a Plane through a Point Parallel to a Line.

1. Pass a line through the point parallel to the given line. (Art. 10a3.)

2. The traces of this line will lie in the corresponding traces of the plane.

Note that there are an indefinite number of solutions, as a line does not determine a plane. (Art. 18.)

41. To Pass a Plane through a Point Perpendicular to a Line.

In Fig. 41a (Plate VII), given the point C and the line AB, while B is the required plane.

1. The traces of the required plane B will be perpendicular to the corresponding projections of the line. (Art. 35d.)

2. Through the given point C, draw a line, CD, parallel to one trace of the required plane (Art. 10a3); *i. e.*, through c^V, draw c^Vd^V ⊥ to a^Vb^V, and through c^H, draw c^Hd^H parallel to the GL.

3. This line, CD, is a line of the required plane B and its trace will determine a trace of the required plane; *i. e.*, d^H, being the HT of the line, is a point on the HT of the plane B.

3a. Through this trace, d^H of the line CD, draw a trace, HT, of the required plane B, perpendicular to the corresponding projection, a^Hb^H, of the given line AB.

3b. Through the point, x, where this trace, HT, cuts the GL, draw the other trace, VT, perpendicular to the other projection, a^Vb^V.

42. To Pass a Plane through a Point Parallel to Two Intersecting Lines.

As two intersecting lines determine a plane, this is the same as Art. 36, or —

Through the point, draw two lines parallel to the given lines and pass a plane through them. (Art. 32.)

43. To Pass a Plane through a Point Perpendicular to Two Intersecting Lines.

Similar to Art. 42 and Art. 37.

44. To Pass a Plane through a Line Parallel to a Line.

Through a point of the line, draw a line parallel to the other line. These two lines determine the required plane. (Art. 32.)

45. To Pass a Plane through a Line Perpendicular to a Line.

This is generally impossible except when the line through which the plane is to be passed is parallel to either trace of the required plane. For instance, if the first line is CD and the second line is AB in Fig. 41a.

46. To Find the Angle between Two Lines and to Bisect It.

In order to bisect an angle, we must see it in its true size. This necessitates revolving the angle into one of the co-ordinate planes. To do this, we must pass a plane through the two lines forming the angle (Art. 32) and revolve this plane into one of the co-ordinate planes. Now, in dealing with a plane, we deal with its traces on H or V. So, then, we have one line of the plane in H or V, *i. e.*, HT or VT. It is then only necessary to revolve the vertex about this line into H or V. (Art. 27.) The angle is now seen in its true size and may be bisected, and after being bisected, it may be revolved back to its original position.

In Fig. 46a1 (Plate VII), the angle is formed by the lines AB and BC.

1. Pass a plane through AB and BC. (Art. 32.) **Note,** one trace is sufficient.

2. Revolve the vertex, B, about this trace into the corresponding plane of projection. For instance, if the HT is used, drop a perpendicular from b^H to HT, or if the VT is used, drop the perpendicular from b^V to VT. In this case the HT was used, then B takes the position b^{RH}. For further details, see Art. 27. (Fig. 46a2.)

3. We now have the angle in its true size, viz., $m^H b^{RH} n^H$.

4. Bisect this angle by the bisector $b^{RH}k^H$, the projections of K being k^H on HT and k^V in GL. (Fig. 46a.)

5. Revolve the bisector, etc., back again. The bisector has its V projection in $b^V k^V$ and its H projection in $b^H k^H$.

46a. Same. One line being parallel to a coordinate plane.

To find the angle ABC and to bisect it, if AB is parallel to a co-ordinate plane; *e. g.*, if AB is parallel to H. Fig. 46b1 (Plate VII).

1. Revolve C about the line AB as an axis until it is parallel to H. One side of the right triangle = $c^V n$, other side = oc^H, and the hypotenuse = oc^{RH}. (Art. 27.)

2. Then $a^H b^H c^{RH}$ = the true size of the angle.

3. Bisect the angle by the bisector $b^H d^{RH}$. (Fig. 46b.)

4. Revolve D back to its position in space. This is done by —

a. Drawing $d^{RH} c^{RH}$ parallel to $a^H b^H$ until it intersects $b^H d^{RH}$.

b. Drawing $c^H d^H$ parallel to $a^H b^H$ until it intersects $d^{RH} r \perp$ to $a^H b^H$.

c. Finding d^V, which is directly above d^H and opposite c^V.

This process may be checked by revolving D about the line AB until it is parallel to H.

Or we may do the problem as follows (Fig. 46c):

1. Join C to any convenient point, A, in AB.

2. Join C revolved (C revolved about AB as an axis until parallel to H) to this point A. The line AC is then revolved, and hence every point in it is revolved.

3. Find the intersection of the bisector BE (of the angle ABC, now shown in its true size $a^H b^H c^{RH}$) and the line AC revolved. This gives the point e^{RH}.

4. e^{RH}, in being revolved back, moves along the perpendicular $e^{RH}p$ ($\perp$ to a^Hb^H) until it intersects a^Hc^H. This gives e^H with e^V directly above e^H on the line a^Vc^V.

47. To Find the Intersection of Two Planes.

This problem occurs in two cases, one where the traces intersect within the limits of our drawing paper (Art. 48), and the other where they do not (Art. 49).

48. Intersection of Two Planes — on Paper.

In Fig. 48a (Plate VII), given the planes A and B.

1. The intersection of two planes is a line.

2. A line is determined by two points.

3. Find the point where the planes intersect on H, V, or P. In Fig. 48a, the VTs intersect at D.

4. Find the point where the planes intersect on one other co-ordinate plane. The HTs intersect at E.

5. Join the points D and E, found in 3 and 4. Be careful to join corresponding projections.

49. Intersection of Two Planes — off Paper.

Given the planes A and B in Fig. 49a (Plate VIII).

1. Same as Art. 48.

2. Same as Art. 48.

3. Same as Art. 48. (VTs intersecting at E.)

4. As this step is impossible by the preceding article, due to the fact that the other traces intersect

off the paper, we must find another point on the line of intersection by

a. Passing an auxiliary plane perpendicular to the co-ordinate plane on which the traces do not intersect. In Fig. 49a, the auxiliary plane is perpendicular to H.

b. The V (or H) traces of the original planes on this new co-ordinate plane, which is parallel to V, will be parallel to the traces of those planes on V. The new traces will be x^Vf^V, parallel to the VT of A, and y^Vf^V, parallel to the VT of B.

c. These new traces will intersect in a point, which gives us a second point on the line of intersection. This point is F, f^V being on the VTs and f^H on the new GL or HT of the auxiliary plane.

5. Same as Art. 48. Join E and F.

EF is the required intersection of the two planes.

49a. Same, when neither HTs nor VTs intersect on the paper.

Fig. 49b shows this case where Art. 49 must be applied twice.

50. To Find the Point in which a Line Pierces a Plane.

In Fig. 50a1 (Plate VIII), given the plane A and the line CD.

1. Pass a plane through the line. Usually a plane perpendicular to one plane of projection is most convenient. In Fig. 50a, plane C is passed through the line CD perpendicular to H. This is a projecting plane. (Art. 11.)

2. Find the intersection of the two planes A and C. (Art. 48 or Art. 49.) This gives us the line GH.

3. Find intersection of the given line CD and the line GH found in 2. (Art. 13.)

4. J is the required point.

Note. The above example may be checked by passing a plane through CD perpendicular to V, etc. This problem is very useful in finding the shadow of a point on an oblique plane. (See text on Shades and Shadows.)

51. The Projection of a Point on an Oblique Plane.

Given the point C and the plane A. (Fig. 51a1, Plate VIII.)

1. Drop a perpendicular, CC′, from the point C to the plane A. (See Art. 35e. Fig. 51a2.)

2. Find the intersection of this perpendicular, CC′, and the plane A. (Art. 50. Fig. 51a.) The auxiliary plane = B, intersection of the two planes = XY, and the required intersection = C′.

3. C′ is the required point.

52. To Find the Projection of a Line on an Oblique Plane.

Project each end of the line on the plane. (Art. 51.)

Join corresponding projections.

In Fig. 52a1 (Plate VIII), the line BC and the plane A are given.

Drop a perpendicular, BB^A, from B to the plane A. Draw $b^Hb^{AH} \perp$ to HT of A and $b^Vb^{AV} \perp$ to VT of A. Find the intersection of the line BB^A and the plane A at B^A, shown by b^{AH} and b^{AV}. (Fig. 52a2.)

Likewise, drop a perpendicular from C to the plane A. This gives C^A (c^{AH} and c^{AV}).

Join B^A and C^A, *i. e.*, join b^{AH} and c^{AH}, b^{AV} and c^{AV}.

B^AC^A is the required projection.

This may be checked by projecting a third point of the line on the plane.

53. To Find the Distance from a Point to a Plane.

To find the distance from C to the plane A in Fig. 51a (Plate VIII).

1. Project the point C on the plane. (Art. 51.) (C′ is the point when so projected.)

2. Find the true length of this projecting perpendicular, CC′. (Arts. 28, 29.) The true length $= c^{RV}c'^{RV}$; c^Vc^{RV} being equal oc^H and $c'^Vc'^{RV}$ equal $o'c'^H$. (Fig. 51a, extra.)

54. To Project a Point on a Line, or to drop a perpendicular from a point to a line.

In Fig. 54a1 (Plate IX), given the point A and the line CD.

1. Pass a plane A through the point A perpendicular to the line CD. (Art. 41. Fig. 54a2.)

2. Find the intersection of this auxiliary plane A and the line CD. (Art. 50. Fig. 54a3.) (The plane through the line is B, the intersection of planes A and B is XY, intersection of XY and CD is A′, which is the intersection of plane A and the line.)

3. A′ is the required projection. (Fig. 54a3.)

55. Another Method for Art. 54.

In Fig. 55a1 (Plate IX), given the point A and the line CD.

1. Pass a plane B through the point A and the line CD. (Art. 34. Fig. 55a2.) Only one trace is necessary, *e. g.*, VT of B.

2. Revolve the line CD and the point A about this trace, VT of B, into the proper co-ordinate plane, — in this case, into V. (Art. 27.) (The line takes the position $x^{RV}y^{RV}$ and the point becomes a^{RV}.) The line and the point are now shown in their true relation. (Fig. 55a3.)

3. Drop a perpendicular $a^{RV}a'^{RV}$ from the point revolved, a^{RV}, to the line revolved, $x^{RV}y^{RV}$. (Fig. 55a.)

4. The point, a'^{RV}, where this perpendicular, $a^{RV}a'^{RV}$, cuts the line revolved, $x^{RV}y^{RV}$, will be the

revolved projection of the point A on the line CD.

5. Revolve this point, a'^{RV}, back into the line. Draw $a'^{RV}a'^{V}$ perpendicular to VT of B until it cuts $c^{V}d^{V}$. This gives a'^{V}. Draw $a'^{V}a'^{H}$ perpendicular to GL until it cuts $c^{H}d^{H}$ at a'^{H}.

6. a'^{V} and a'^{H} are the required projections of A on CD.

56. To Find the Distance from a Point to a Line.

In Figs. 54a and 55a (Plate IX), A is the point and CD is the line.

1. Project the point on the line. (Arts. 54 or 55.) This gives A′.

2. Find the true length of this projecting line AA′. (Arts. 28, 29.) The true length is $a^{RV}a'^{RV}$.

Note in Fig. 55a that the true length had previously been found, while in Fig. 54a the true length is indicated by the dot and dash lines.

57. To Draw Two Lines Perpendicular to Each Other.

See Art. 10a4.

1. Draw one line CD at random. (Art. 12. Fig. 54a1 or 55a1 [Plate IX].)

2. Assume a point in space outside of the line, *e. g.*, point A.

3. Project this point A on the line CD, or, in other words, draw from the point A a line AA′

perpendicular to the given line CD. (Arts. 54 and 55. Figs. 54a3 and 55a.)

4. This gives the line AA′ perpendicular to the given line CD.

58. To Find the Distance between Two Lines (not in the same plane), or to erect a perpendicular to two lines.

In Fig. 58a1 (Plate IX), given the lines AB and CD.

1. Through one line AB pass a plane A parallel to the other line CD. (Art. 44.) This, you recall, is done by drawing through any point, B, of the line AB, a line EF parallel to CD and passing a plane A through AB and EF. (Fig. 58a2.)
2. The line CD is parallel to the plane A passing through the line AB, and the distance from the one line CD to the other line AB is equal to the distance from the one line CD to the plane A passing through the other line AB, and is found in step 3.
3. Find the distance from any point K of the line CD to the plane A. (Art. 53.) KL is ⊥ to the plane, L is the point where the line KL intersects the plane, and $K^{RH}L^{RH}$ is the true length of KL. (Fig. 58a.)
4. $K^{RH}L^{RH}$ equals the required distance.

59. To Find the Angle between a Line and a Plane. (Common Method.)

In Fig. 59a1 (Plate X), given the line AB and the plane A.

1. Through any point A of the line AB drop a perpendicular to the plane A. (Art. 35e.) AM is this perpendicular.

2. Find the true angle between these two intersecting lines MA and AB. (Art. 46.) HT of B is the HT of the auxiliary plane, a^{RH} is A revolved into H, $m^{H}a^{RH}n^{H}$ is the angle. (Fig. 59a.)

3. $m^{H}a^{RH}n^{H}$ is the complement of the required angle. Make $m^{H}a^{RH}x$ equal 90°, then $n^{H}a^{RH}x$ is the required angle.

59a. Another Method for Art. 59.

In Fig. 59b1 (Plate X), given the line AB and the plane A.

1. The required angle is the angle between the line AB and its projection A′B′ on the plane A.

2. Project the line AB on the plane A. (Art. 52.) This projection is A′B′. (Fig. 59b2.)

3. Find the true angle between these two lines AB and A′B′ intersecting at C. (Art. 46.) The auxiliary plane is shown by its HT, and c^{RH} is the revolved position of the vertex C, while $x^{H}c^{RH}z$ is the true angle required. (Fig. 59b.)

60. To Find the Angle between Two Planes.

Method I. In Fig. 60a1 (Plate X), given the two planes A and B.

1. Assume some point A between the two planes.

2. From this point A drop a perpendicular to each plane. (Art. 35e.) AC is perpendicular to A and AB is perpendicular to B. Note that it is not necessary to find the intersection of the perpendicular and its plane.

3. Find the angle between these two perpendicular lines AC and AB. (Art. 46.) Auxiliary plane is shown by its HT, and $y^H a^{RH} x^H$ is the angle found. (Fig. 60a.)

4. $y^H a^{RH} x^H$ is the supplement of the required angle. $x^H a^{RH} z$ is the required angle.

Method II. In Fig. 60b1 (Plate XI), given the planes A and B.

1. Pass a plane C perpendicular to the intersection AB of the two planes. (Art. 35.) VT of C is perpendicular to $a^V b^V$ and HT of C is perpendicular to $a^H b^H$. (Figs. 60b2 and 60b.)

2. This plane C cuts two intersecting lines XY and YZ from the two given planes A and B. Points X and Z are readily found in Fig. 60b2.

3. In order to find the point Y, find the intersection of the auxiliary plane C and the intersection AB of the two given planes. (Art. 50.) WU is the intersection of the auxiliary plane C and the projecting plane through AB, and WU intersects the line AB in the point Y. (Fig. 60b.)

4. Find the true size of the angle XYZ, formed

by XY and YZ. (Art. 46.) This gives the angle $x^V y^{RV} z^V$.

Method III. Same as Method II except steps 3 and 4, which are as follows:

3. To find point Y —

a. Revolve AB into V. This gives $a^V b^{RV}$. (Fig. 60c3, Plate XI.)

b. Drop a perpendicular $w^V m$ from w^V, where VT of C cuts $a^V b^V$, to $a^V b^{RV}$. This represents the plane of the angle XYZ revolved into V. (Fig. 60c4.)

c. Revolve the vertex, m or $y^{RV'}$, back into space. (Draw $my^V \perp$ to $a^V b^V$, which gives y^V. y^H is directly below and on $a^H b^H$.) (Fig. 60c.)

4. Find the true size of this angle XYZ. (Fig. 60c.)

a. With w^V as a center, and a radius $w^V y^{RV'}$, describe the arc $y^{RV'} y^{RV}$ to the line $a^V b^V$, giving y^{RV}.

b. $x^V y^{RV} z^V$ is the required angle.

Note. Sometimes step 3 is done on a separate drawing. (See Fig. 60d.) This represents a side or profile view taken along $a^V b^V$. It explains itself.

61. To Find the Angle between a Plane and H or V.

In Fig. 61a (Plate XI), given the plane A and the co-ordinate plane H.

Same analysis as Art. 60, Method II.

1. Plane B is perpendicular to the intersection of planes A and H (note that the intersection of planes A and H is the HT of A), the HT of B being perpendicular to HT of A and VT of B being perpendicular to the GL.

2. This plane B cuts the two intersecting lines XY and YZ from the two given planes, XY being the intersection of planes A and B, and YZ being the intersection of planes B and H.

3. The vertex Y is the intersection of the auxiliary plane B, and the intersection of A and H, *i. e.*, the HT of A, and consequently is where the HT of A and HT of B intersect.

4. The true size of the angle XYZ is $x^{RH}y^{RH}z^{RH}$.

In a similar manner, the angle between the plane A and V might be found.

61a. Other Methods for Art. 61.

Of course, this problem may be worked out by other methods. (Art. 60.)

For instance, Fig. 61b (Plate XI) shows the same example by Method I.

The given planes are A and H.

1. The assumed point is A.

2. AB is perpendicular to plane A, and AC is perpendicular to H.

3. Find the angle BAC between AB and AC.

4. In the case of H or V, the supplementary angle is not necessary. $b^{V}a^{RV}c^{V}$ is the required angle.

62. Corollary to Art. 61.

Given either trace, VT, of a plane A, and the angle $b^{RV}a^{V}b^{V}$, between the plane A and the other co-ordinate plane, H, to find the other trace, *i. e.*, the HT of the plane A. (Fig. 62a, Plate XI.)

1. Through any point A (a^{V}), of the given trace, VT, draw two lines, one, $a^{V}b^{V}$, perpendicular to the given trace, and the other, $a^{V}b^{RV}$, making the required angle, $b^{RV}a^{V}b^{V}$, with this perpendicular.

2. Revolve the plane of these two lines, $a^{V}b^{V}$ and $a^{V}b^{RV}$, about the former, $a^{V}b^{V}$, until perpendicular to the corresponding trace of the given plane. b^{RH} will take the position b^{H}.

3. The position of b^{H}, as found in 2, will be a point on the required (H) trace.

63. Miscellaneous.

Similar to Arts. 61 and 62, numerous corollaries might be inserted; for instance, given one plane and the angle it makes with another, etc.

The student is left to work them out by himself as necessity demands.

64. The Projection of a Circle on the Co-ordinate Planes.

In Fig. 64a (Plate XI), the diameter of a circle as well as its center, A, lying in a known plane A, is given. To find its projections on H and V.

1. Revolve the center A about one trace (HT) of the given plane A into that (H) co-ordinate plane. The center takes the position a^{RH}.

2. Draw the circle d–e–f–g as required with a^{RH} as the center.

3. The projections of the circle will be ellipses. In the H projection, $d^H f^H$ is equal the major axis, and the minor axis is $e^H g^H$, which is obtained by drawing g–d–j and f–e–k (all in H) and also jd^H and kf^H, thereby locating e^H and g^H on the minor axis.

4. Draw the ellipse, using the method of trammels.

5. In a similar manner, the other (V) projection may be secured.

Plate I

Plate II

Plate III

Plate IV

Plate V

Plate VI

Plate VII

Plate VIII

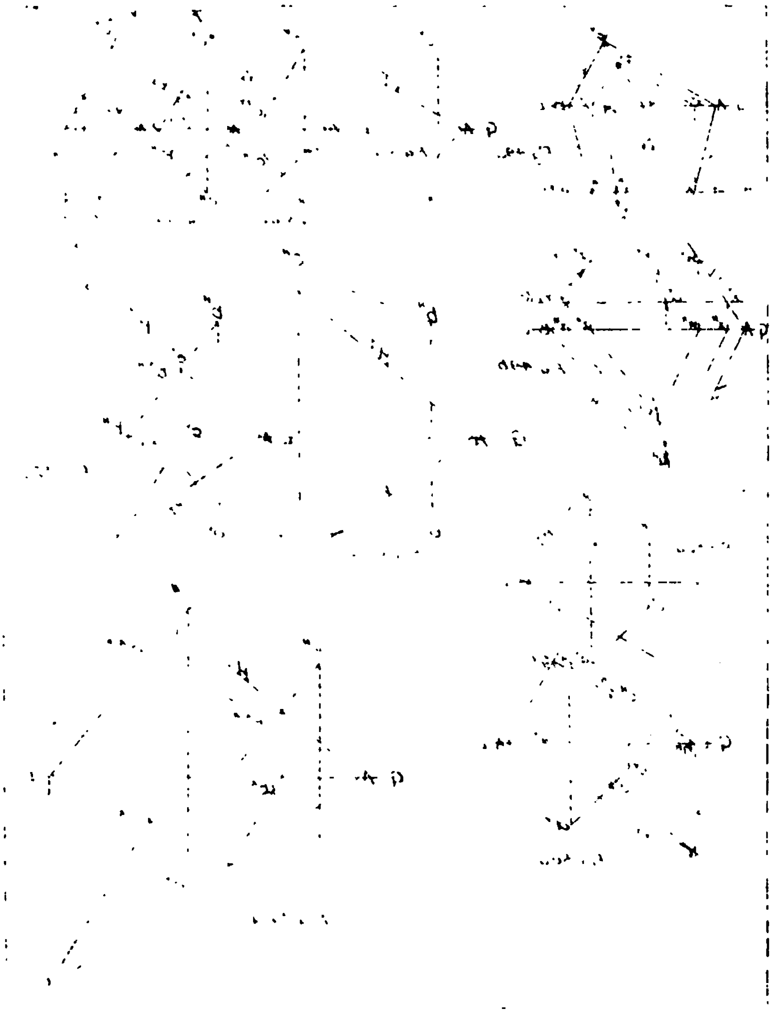

Plate IX

Plate IX